Two Dimensional Wing and Blade Mathematical Theory

Detailing and Extending Material in Standard References

Wind Turbine Blade Section Thickness
Part 4
Aerodynamic Lift On Blades Now Discussed
The Kutta-Joukowsky Condition

"It is amazing that today, ...100 years after the first flight of the Wright Flyer, groups of engineers, scientists, pilots, and others can gather together and have a spirited debate on how an airplane wing generates lift. Various explanations are put forth, and the debate centers on which explanation is the most fundamental." quote by John D. Anderson, Jr., author of *Introduction to Flight*, McGraw-Hill, ISBN 0-07-299071-6, Curator of Aerodynamics at the Smithsonian Institution's National Air & Space Museum, and Professor Emeritus at the University of Maryland.

Anthony Chessick, IntegEner-W, 2009

If it wasn't clear before, it should be by now that other considerations apply to these analyses than those presented in the referenced book[1] and, by extension, those in other books on this subject as well. The rigor involved in creating math-oriented computer programs based on the subject material, a technology that was not so readily available when the book was written, has caused new light to be shed on it, some of which even disputes the conclusions reached within as those arrived at somewhat peremptorily. Beyond this, the requirements of wind energy are placing renewed emphasis on understanding the details of the logical arguments.

Limitations of Flow Kinematics

The descriptions of flow as described using the ψ and ϕ functions have primarily been of a kinematical nature, that is, without reference to the mass of the fluid. The fluid is constrained to follow certain paths when bending around objects in its path due merely to its being a fluid. It was found that mathematical descriptions of the kinematic-based flow could be made more complete by means of superposing two or more elementary flow descriptions together to obtain a final flow description. What is not guaranteed, however, is that all necessary elements have been in fact included in the superpositions, thereby possibly rendering the flow description obtained not entirely accurate.

In particular, a common failing of kinematics is when division by zero occurs, implying infinite velocities at some location, such as that at the center of a vortex or, as is seen in the text at hand, the flow bending around the sharp point of the trailing edge of an airfoil. To treat the word "infinite" with so little appreciation for its meaning while moving from one math step to the next is not in keeping with practical reality. It is here where some diversion must be taken from the treatment found on the pages of the referenced book as copied into Part 1 of this series earlier.

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[1] Refers to Abbott, Ira H. and von Doenhoff, Albert E., *Theory of Wing Sections*, 1959, ISBN: 486-60586-8, Dover Publications, New York, Chapter 3, Article 3.5, pages 50-53.

Flow Circulation as Described Using the Flow Element of Vortex Motion

The words "infinite velocity" were used earlier in the reference book during the development of flow theory at locations such as the centers of sources and sinks and the centers of vortices. Since these locations are eliminated when considering bodies of finite dimensions surrounding them, nothing more was said. The more general implications, however, are not so readily dismissed. In the case of vortices, the calculation of how much energy is necessary to create a vortex that is to extend from its center out into an infinite region of space, even considering its declining velocity with distance, would be, in fact, infinite as a simple integration of the kinetic energy over space in polar coordinates makes clear:

$$E = \frac{1}{2}mv^2 = \frac{1}{2}\rho \int_0^\infty v^2 \, dA$$

Since $v = \dfrac{\Gamma}{2\pi r}$ and $dA = 2\pi r \, dr$

$$E = \frac{1}{2}\rho \int_0^\infty \left(\frac{\Gamma}{2\pi r}\right)^2 2\pi r \, dr = \frac{\rho\Gamma^2}{4\pi} \int_0^\infty \frac{\cancel{r}}{r^{\cancel{2}}} \, dr$$

$$= \frac{\rho\Gamma^2}{4\pi}\Big[\ln r\Big]_0^\infty = \frac{\rho\Gamma^2}{4\pi}(\infty - (-\infty)) = \frac{\rho\Gamma^2}{2\pi}(\infty)$$

That is why vortices are normally incompletely realized over extended distances in practice, circulations and swirls that have limited dimensions, unlike the indefinite dimensions assumed in these equations. In addition, to say that the generation of a flow circulation vortex is a necessary part of the creation of aerodynamic lift by a wing or a blade may be implying quite a chore from the standpoint of the energy requirement for doing so that is not being given proper attention in these earlier analyses.

Another concession to reality must be made. Since atmospheric pressure is the only term in the Bernoulli Equation driving the velocities for air, the maximum velocity that air can achieve as accelerated by it, not commonly understood, is about 910 mph, fast certainly but not infinite.

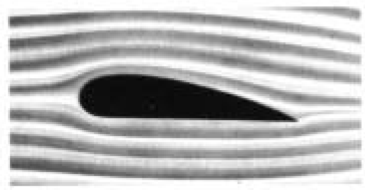
The Kutta-Joukowski Condition as Determining the Flow Circulation

The Kutta-Joukowski Condition, briefly described and as presented in the reference text, is the fixing of the airflow circulation, Γ, to that value which eliminates the flow occuring around the sharp point of the trailing edge, giving rise to infinite velocities in the flow mathematics.

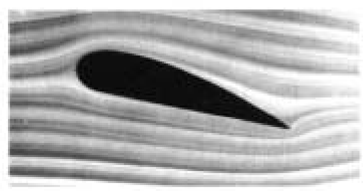
For all the reasons given above and also from the standpoint that an infinite velocity at the trailing edge falls out of these equations, as it does in those of other physical circumstances, as occupying only an infinitesimally small volume thereby maintaining an overall momentum within finite bounds, questions may be raised about the limitations of this well-known theory in determining the flow circulation. The momentum can be proven to be finite over a small region extending out to R from the trailing edge as follows:

$$M = mv = \rho \int_0^A v\, dA = \frac{\rho\Gamma}{2\pi} \int_0^R \frac{2\pi \cancel{r}}{\cancel{r}}\, dr$$
$$= \rho\Gamma \Big[r \Big]_0^R = \rho\Gamma R$$

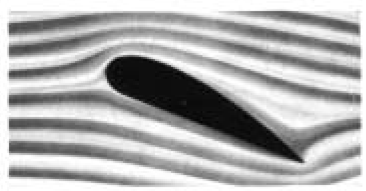
Here below is an image of an airfoil and its flow lines taken from a page in a higher education physics textbook in the section treating aerodynamic lift[2]. Although provided for other reasons and not intending to do so, the image displays flow that does not satisfy the Kutta-Joukowski Condition in any of the three angles of attack portrayed, leaving the airfoil not quite exactly at the trailing edge:



(a)



(b)



(c)

No mention is made of the source of this image nor how obtained. One reason for the lack of smooth flow off the trailing edge may be the relative thickness of the airfoil, about 19.4% here, about which more will be said below.

One answer to the dilemma of what happens at the trailing edge may be that a small circular flow rotation is caused to occur just above it that could be superposed mathematically into the general flow description for more accuracy. This small "air wheel" then serves to act as if it were a part of the airfoil itself thereby effectively "rounding" the trailing edge so it is seen to be not as sharp by the rest of the flow.

Obtaining a Correct Value for the Flow Circulation

If the Kutta-Joukowski Condition is not used for determining the correct value for the flow circulation, then how may the correct value be obtained? This is the question that must be asked

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[2] Sears, Francis Weston, *Mechanics, Wave Motion, and Heat*, 1959, LOC# 58-5058, Addison-Wesley, Reading, Massachusetts and London, England, Chapter 15, Firgure 15-18, pg. 408.

when moving from a purely kinematical view of the flow to one of practical significance. In general terms, another equation is needed that brings Newton's Law into the picture, as does, for example, the Navier-Stokes Equation, which is in the dimensions of force vector distribution per unit volume. Suffice it to say that in so doing the problem becomes one requiring effort that goes beyond the purposes being pursued here.

That is not to say that an intuitive grasp can not be had of the picture and one that satisfies the need to understand what must be done in general in designing the airfoil profile.

The description of the airfoil as a circular cylinder, used to begin the analysis of the stream and cross stream flow lines, was a convenient geometry that allowed the math to develop. When the flow circulation was added, a difficulty arose in that no mechanism for the creation of the circulation was provided. It was simply assumed to exist. The equation presented for the value of the circulation,

$$\Gamma = 4\pi a V \sin\theta$$

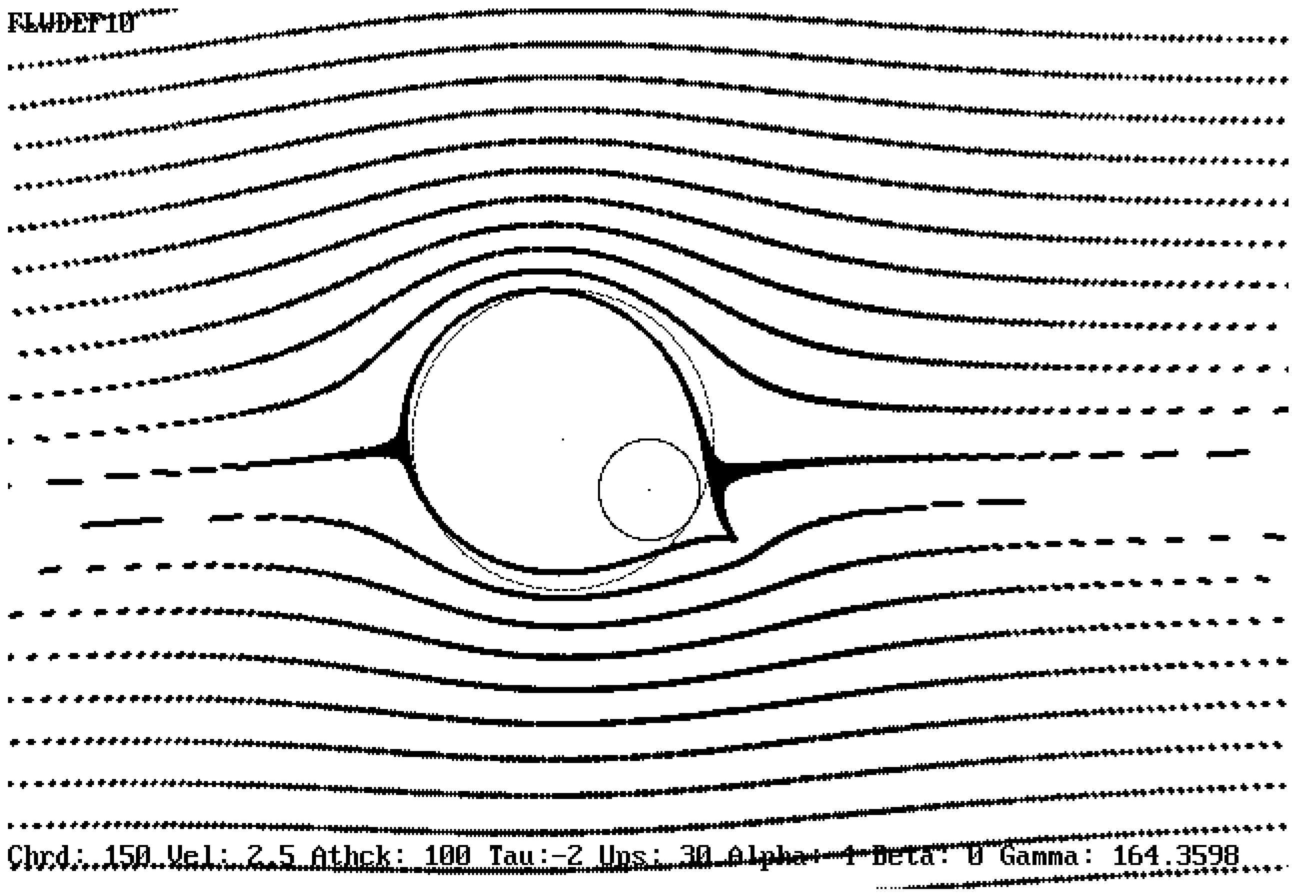
was not an identification of the physical source giving rise to the circulation but only a statement providing the relationship between the circulation and the value of the angle, θ, on the circular cylinder of the flow stagnation points below the horizontal axis. Again, many earlier efforts fail to provide a value for the circulation and it remains a parameter whose value must be set by some other means.

In the study of aerodynamics, reliance was once placed on the Magnus Effect to provide the flow circulation, accomplished by simply rotating the cylinder, but this was found in the case of airfoils to have an effect of low significance, i.e., the spinning airfoil did not create a large measure of flow circulation in the air surrounding it.

For static, nonrotating airfoils, then, no flow circulation is available and something must be done to create it. The Kutta-Joukowsky Condition presumes to create a flow circulation by requiring the flow to leave the trailing edge smoothly, with no peripheral flow folding around it. If the airfoil has no trailing edge as is the case with the circular cylinder, then, again, no circulation can be provided, even so. If it is modified while still retaining most of its circular shape to have a small trailing edge then some tendency exists for the flow to leave the airfoil nearer it as the angle of attack changes. This process continues as the airfoil is made thinner and longer, with a more prominent and sharper trailing edge. Eventually when the airfoil is thin and long enough the flow leaves the airfoil at the trailing edge for all required angles of attack.

This can be seen also in the creation of a pitching moment in the airfoil. If a circulation in the flow is created, then the airfoil must see a reaction to it and this reaction is a pitching moment. For a positive circulation resulting in a lift force in the upward direction this would be a pitching moment in the downward direction, i.e., reducing an upward angle of attack of the leading edge. The existence of a pitching moment, then, assures a flow circulation is being created. The Kutta-Joukowsky Condition, absent the equations of motion, does not provide information on how a pitching moment is obtained.

Here is a run of a computer program, FLWDEF10, written in BASIC, presenting the flowlines for a thick, blunt, almost round cylindrical, "blade" pitched to 30 degrees with a small trailing edge. The flow can be seen to pass by the trailing edge with only a minor, localized effect:



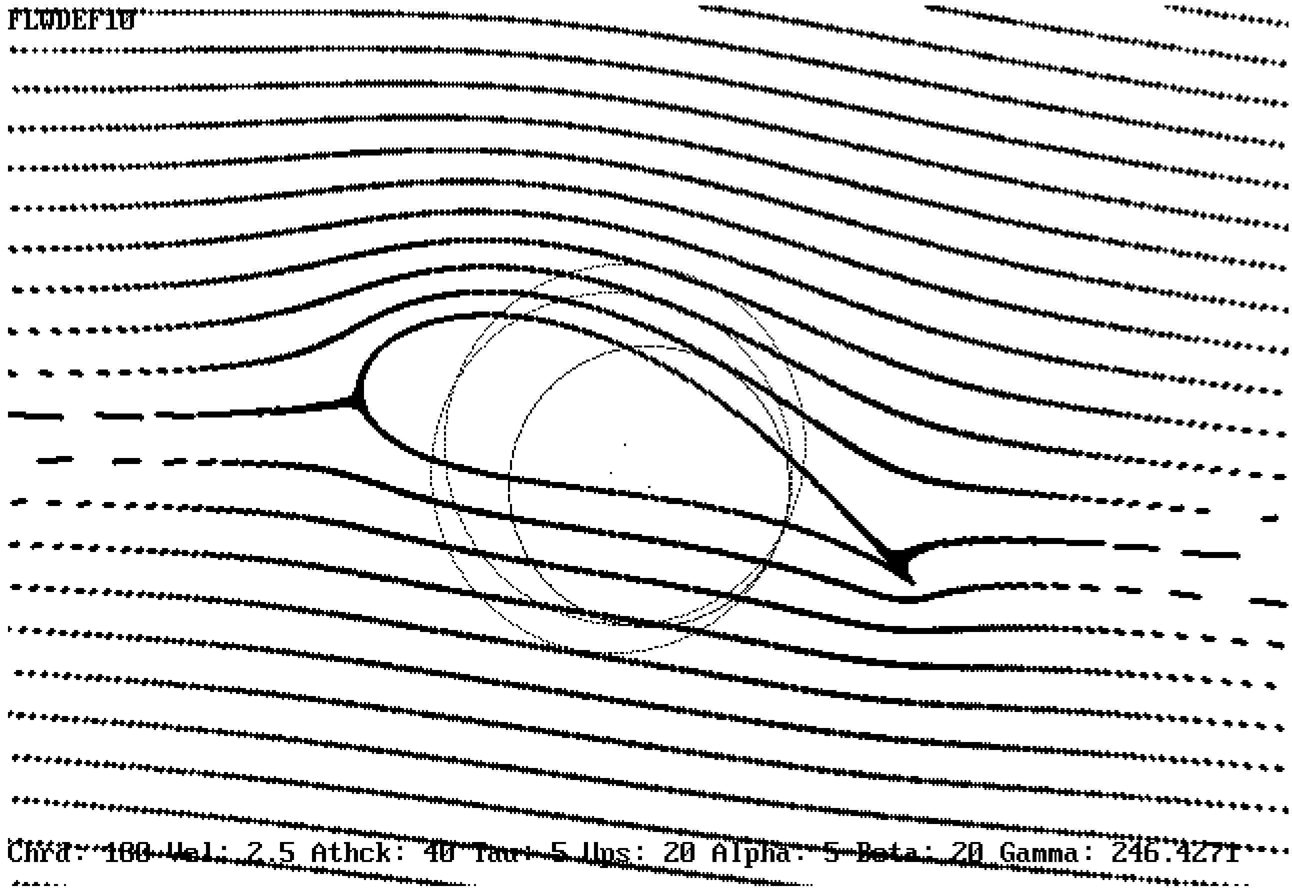
Recognizing this trailing edge, however, a small circulation angle, alpha, value of 4 degrees was entered, dropping the flow impact stagnation points on the right and the left down from the horizontal a short distance as may be noted. As the blade thickness is decreased in further computer runs of this type, it is intuitively clear that larger circulation angles would be appropriate in satisfying equations of motion, eventually reaching the point where the flow leaves the blade at the trailing edge.

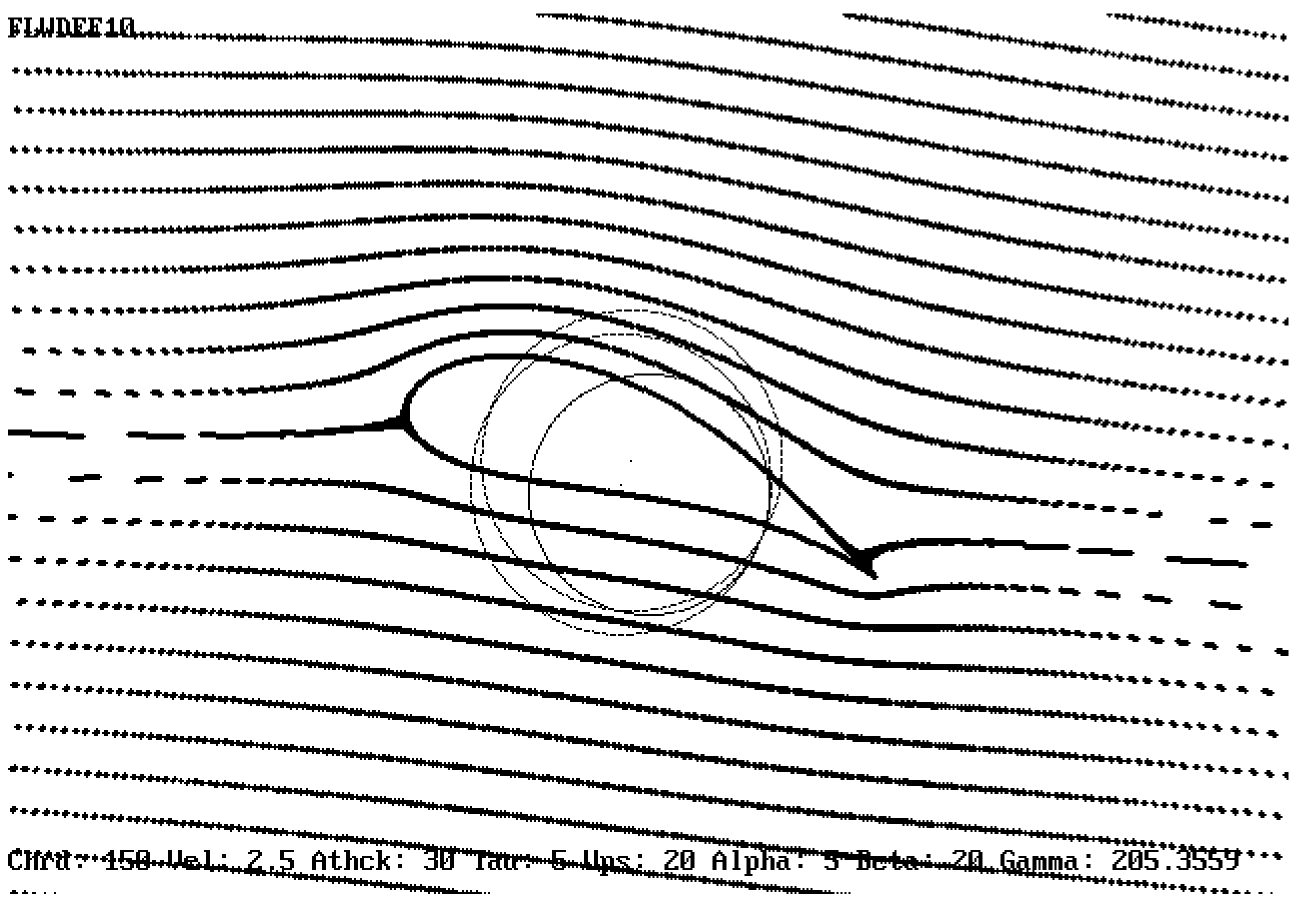
The Lift Coefficient According to the Referenced Text

The referenced text then goes on to give some thought to how blade thickness affects the lift coefficient. The conclusion reached is that "for wing sections approximately 12 percent thick, the theoretical lift-curve slope is about nine percent greater than its limiting value for thin sections". By this is meant the slope of the coefficient of lift vs. the angle of attack curve.

The arguments made for this important conclusion are brief and can be said to be not convincing.

In particular, a careful look in detail at the logic presented reveals that the blade chord is being increased along with the blade thickness while making an assumption that the blade chord is remaining constant. Thus a larger blade cross section including chord will have more impact on the airflow and a greater lift coefficient, which is not in dispute. What is a question to be raised is that the factor ($a + \varepsilon$) is being divided by a to result in the factor ($1 + \varepsilon/a$) and should more accurately be being divided by ($a + \varepsilon$) instead to result in the factor of 1. Doing so would eliminate an effect on the lift coefficient by the blade thickness independent of changes to the blade chord. The next two runs of FLWDEF10 illustrate this point:


FLWDEF10
Chrd: 100 Vel: 2.5 Athck: 40 Tau: 5 Ups: 20 Alpha: 5 Beta: 20 Gamma: 246.4271


FLWDEF10
Chrd: 150 Vel: 2.5 Athck: 30 Tau: 5 Ups: 20 Alpha: 5 Beta: 20 Gamma: 205.3559

A thin blade section of chord 150 and airfoil thickness 10 under identical conditions in the above computer runs, if given a greater relative thickness according to the reference, would look like the first while, if given greater relative thickness under guidelines suggested herein, would look more like the second. For the same circulation alpha, the first has a larger value of Γ due to the larger chord and the second has a value of Γ that would be the same.

The Lift Coefficient Concluded

Finally, a computer run of FLWDEF10 of such a thin blade section was made and copied in below. The circulation angle alpha was adjusted upwards from 5, as in the above two runs, to 12.5, in accordance with arguments presented herein. The flow leaves the blade about at the trailing edge and the value of the circulation Γ has increased to 509.9739, thus increasing the lift coefficient. The flowlines display the characteristic change in direction from left to right that would be expected from high values of lift force creation.