

# Two Dimensional Wing and Blade Mathematical Theory Detailing and Extending Material in Standard References

## Wind Turbine Blade Section Thickness

### Part 3

#### Several Aspects of the Reference's Article 3.5<sup>1</sup> Discussed Flow Fields Converted Using the Joukowski Transformation and the PC BASIC Software

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As mentioned in Part 1 of this series, detailed scrutiny and comment are being presented on material in this textbook reference relating to wind turbine blade section thickness. Some clarification is provided to the analysis in the book and, thereby, questions surface about the conclusions that are drawn therein. If blade section thickness by itself has an effect on the lift force then how may this effect be described.

#### The Joukowski Transformation Put To Use

Up until this point, the "airfoil" being studied for its effect on the flow field is only the circular cross section of a cylinder and it is possible now to convert such an airfoil to an airfoil that is properly thin and tapered with a rounded leading edge and a sharp trailing edge including cross section camber using the Joukowski Transformation previously introduced in Part 2. Such a conversion also converts the entire flow field surrounding the airfoil cylinder that has been obtained from a study of inviscid flow with the properties of continuity and irrotationality developed earlier in Chapter 2 of the same reference. (It is assumed and not proven, incidentally, that the flow properties of continuity and irrotationality are retained in the conversion, a point that may be mentioned here for possible investigation some other time.)

In any event, all would go well in so doing but for the accumulation of mathematical complexities that occur. This very fact is mentioned even in the subject textbook reference on page 52 in commenting on equations 3.9 and 3.10 as was copied into and may be seen in Part 1 of this series.

Computer software to render such complexities amenable to solution and further analysis on the PC was not available when the book was published but is available now. So some time was taken to write a series of brief programs in the BASIC PC language to do exactly what was suggested in the book but left unfinished then.

Here again is the Joukowski Transformation conformal mapping algorithm:

$$\zeta = z + \frac{a^2}{z}$$

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<sup>1</sup> Refers to Abbott, Ira H. and von Doenhoff, Albert E., *Theory of Wing Sections*, 1959, LOC 60-1601, Dover Publications, New York, Chapter 3, Article 3.5, pages 50-53.

In polar coordinates and using De Moivre's Theorem, this algorithm is as follows:

$$\zeta = re^{i\theta} + \frac{a^2}{r}e^{-i\theta}$$

### The Flow Equations in Complex Variable Format

The flow equation for a circular cylinder in complex variable format as presented in the textbook as equation 3.7 is:

$$w = V\left(z + \frac{a^2}{z}\right) + \frac{i\Gamma}{2\pi} \ln \frac{z}{a}$$

The complex variable,  $w$ , is a combination of the stream flow and cross stream flow and is defined as:

$$w = \phi + i\psi$$

where  $\phi$  is the cross stream function and  $\psi$  is the stream function.

In making use of the Joukowski Transformation, the circle cross section must be offset by a distance,  $\epsilon$ , along the x axis, resulting in the following adjustments to the above equation:

$$w = V\left(z^* + \epsilon + \frac{(a + \epsilon)^2}{z^* + \epsilon}\right) + \frac{i\Gamma}{2\pi} \ln \frac{z^* + \epsilon}{a + \epsilon}$$

Then to allow the airfoil to tilt, the equation must be further adjusted as follows as copied in from equation 3.10 of the textbook:

$$w = V\left[(z + \epsilon)e^{-i\alpha_0} + \frac{(a + \epsilon)^2 e^{i\alpha_0}}{z + \epsilon}\right] + \frac{i\Gamma}{2\pi} \ln \frac{(z + \epsilon)e^{-i\alpha_0}}{a + \epsilon}$$

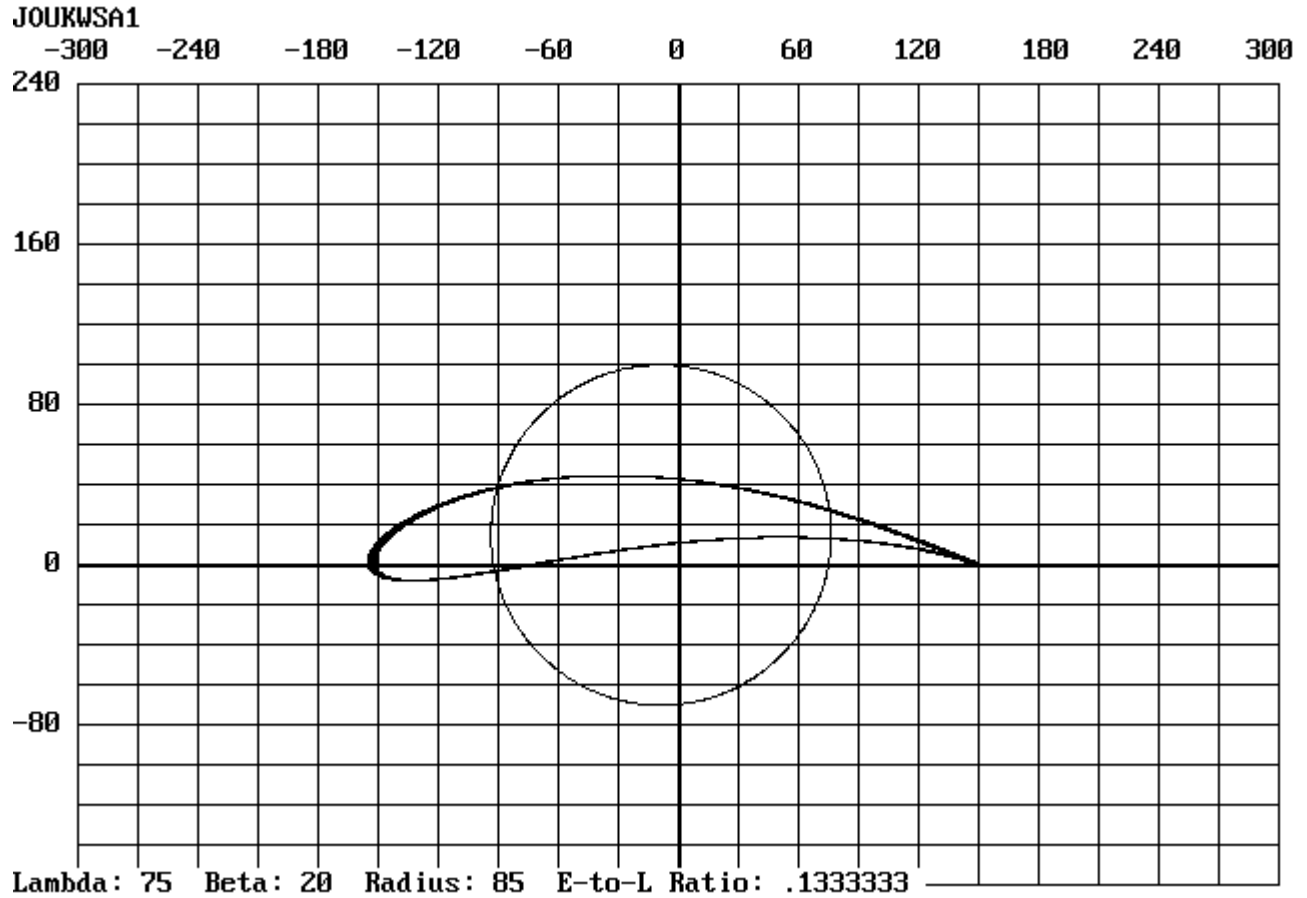
Further, it is possible and even necessary to add the further adjustments of allowing the airfoil and the flow stream to tilt independently of each other and to add camber to the airfoil, meanwhile remembering that the circulation,  $\Gamma$ , may vary as well.

These equations must then be substituted into the transformation algorithm given above to convert the airfoil from a circular cylinder to the thin, tapered Joukowski airfoil.

This entire process was programmed into the PC using BASIC and the results are herewith copied in below as output screens for one matching before-and-after set of airfoil and flow dimensional variables.

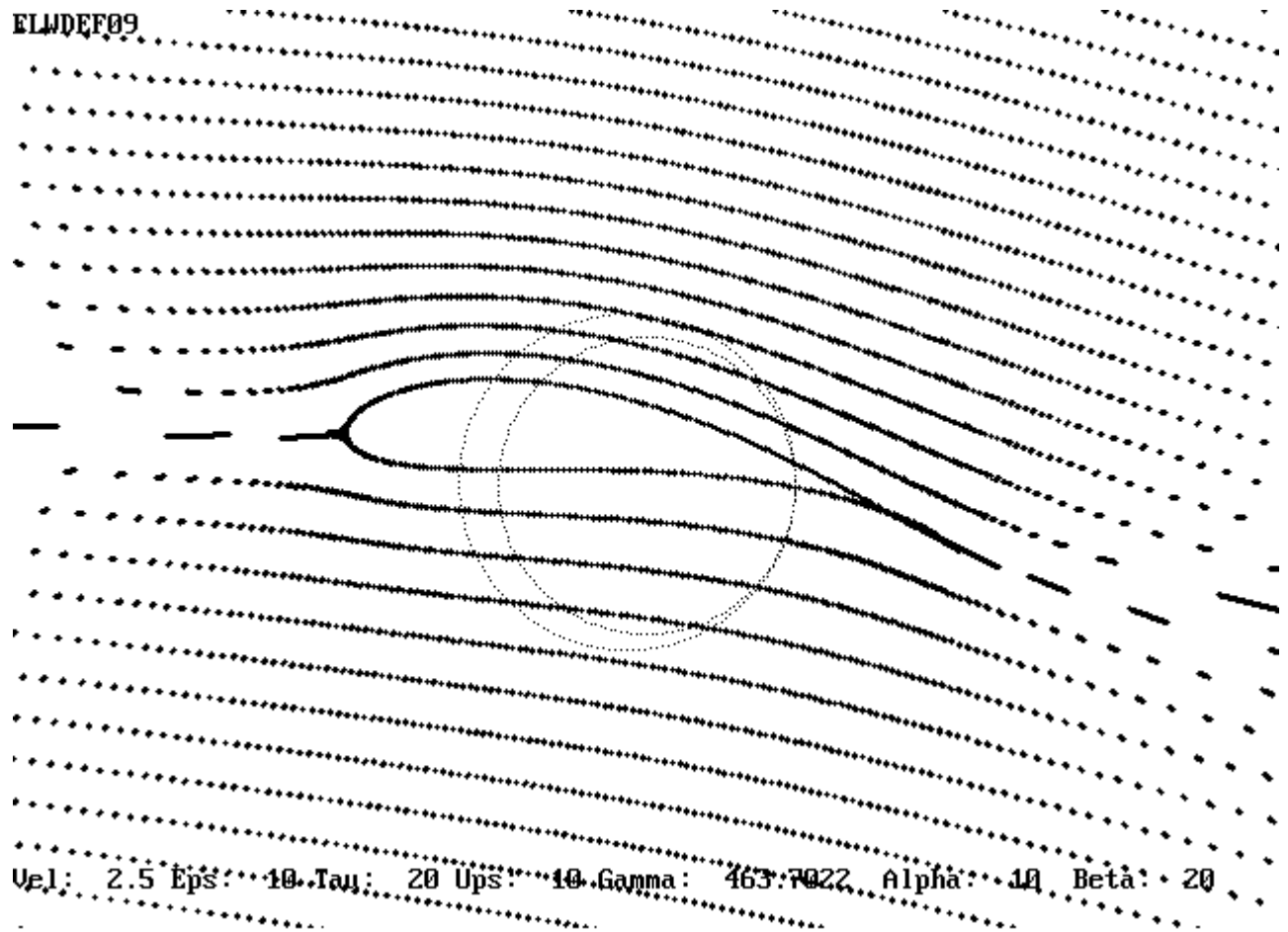
## PC BASIC Output Screens

An airfoil as originally drawn with program input resulting in a specified thickness and camber (using JOUKWSA1):



The "E-to-L" ratio is the difference between the circle radius and lambda divided by lambda and is a measure of the blade thickness. The actual thickness-to-chord ratio is slightly different and is about 14% here. The camber angle, beta, is 20 degrees and is measured at the trailing edge as the angle of the blade mean line above the x axis.

The same airfoil as drawn with program input now including flow stream lines with separate, independent tilt of the airfoil and flow lines and superposed circulation on the flow field (using FLWDEF09):



The cross stream lines would make the drawing unnecessarily detailed and are left out. The airfoil tilt angle,  $\epsilon$ , is 10 degrees, the flow tilt angle,  $\tau$ , is 20 degrees, the circulation impact angle,  $\alpha$ , is 10 degrees, and the camber is the same as that previously, 20 degrees. The actual circulation, which has dimensions of length squared per unit time, is calculated by the program and here is found to be 463.7022, which also is a measure of the lift force produced by the airfoil (and at a tilt angle, itself, from the vertical the same value as that of the flow from the horizontal, 20 degrees, here).

### Airfoil Thickness

As is typical in most drawings of airfoils, the thickness dimension is roughly in the range of about a 10 to 15 percent T/C ratio. This is not to presume that this is the best thickness for all applications and, in fact, for wind energy purposes, some study may be necessary as the technology develops of what may be found as thicknesses that vary from this traditional standard. Comments on airfoil thickness continue in this series.