Two Dimensional Wing and Blade Mathematical Theory
Detailing and Extending Material in Standard References

Wind Turbine Blade Section Thickness
Part 1

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Abstract

A series of short papers discussing several aspects of the text material in a portion of Article 3.5 on pages 50 -53 of the 1959 edition of *Theory of Wing Sections* by Abbott and von Doenhoff are hereby presented beginning with this Part. What follows herein is limited to an overview of the subject of blade section thickness and a copy of the pages themselves, as they relate to this subject. Succeeding Parts in this series present analysis more detailed. The material is considered to be of some interest and relevance to ongoing questions of blade design and so time is taken for such detailed scrutiny and comment.

Overview

The heart of science is a questioning attitude. It was once said of George Bernard Shaw (1856 -1950), the Irish playwright, about his thoughts on scientific matters that he had a "natural, almost effortless grasp of the common sense skepticism which is the lifeblood of scientific advance". He is known to have written, "I have always the right to treat....logical principles as a *reductio ad absurdum*"[1]. It is with this frame of mind that this series of notes is being presented.

Wind turbine blades have thickness as one of their profile dimensions. Why are they such-and-such thick and not some other thickness? This question is bound up intimately with the theories that have surrounded aerodynamics ever since the inception of flight and the twists and turns it has experienced over the years. Now with the advent of wind energy, the question takes on a new importance and we are left to decide if what the industry is left with as the last word that has come down on this subject is, in fact, what is to be the final say on the matter.

The early chapters of Abbott and von Doenhoff, covering as they do with detailed mathematics the classical fluid dynamics theories, are still taken as sacrosanct and inviolate. The logical development can be likened to an act at the circus where a horse is trotting around a ring with two riders, one on top of another standing while riding on top of it, both backwards with the topmost juggling three balls and balancing a stick on his head that supports a saucer spinning on top of it. When they have set up all this motion and have it running, the ringmaster tosses up a penny that lands in this saucer. The audience is so thrilled with the action that the penny is collected and accorded value far in excess of its worth.

The detail on the pages of the book, in other words, seems to have something to do with its credence. At the point where its complexity goes just beyond what most readers can follow then and only then is something of significance declared and accorded acceptance.

Still, a fluid dynamics that follows the flow streamline by streamline throughout a large region of space and through a turn or speed change maneuver as controlled by a set of boundaries while satisfying all the laws of physics relating to pressures and flow directions and velocities is to be given no little respect and admiration. It is just wished as would have Shaw that all the absurdities be removed from it and that it be allowed to shine forth as truth that is as simple as it is profound.

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[1] Quotations from Eric Bentley, *Bernard Shaw*, 1957 Amended Edition, New Directions Paperbooks, New York, pages 71 and 81.

The Text of the Book

The material to be studied herewith is presented below. Blades and wings, whose profiles are drawn or surmised, have generally been designed to have a certain thickness as a percentage of their chord length and that percentage is not far from what is concluded from what can be seen therein, about 12 percent. If this material in this engineering textbook along with similar material in other such books is responsible for this thickness ratio then it has an impact far beyond what would otherwise be expected from just the few short pages on which it is written.

For background on understanding the equations included it would be worthwhile first studying Chapter 2 and early Chapter 3 of the book and also the ten part series of papers under "Two Dimensional Wind and Blade Mathematical Theory" on this reference material presented in offerings of IntegEner-W as may be especially seen on the IntegEner-W website.

For those who are unfamiliar with the Greek alphabet, here are the names of the Greek letters to be seen in the formulas and equations below:

Α α alpha	Η η eta	Ρ ρ rho
Γ γ gamma	Θ θ theta	Φ ϕ phi
Ε ε epsilon	Ξ ξ xi	Ψ ψ psi
Ζ ζ zeta	Π π pi	

Here then is how the thickness-to-chord ratio of 12 percent has been derived:

3.5. Transformation of a Circle into a Wing Section. A circle can be transformed into a shape resembling that of a wing section by substitution of the variable

$$\zeta = z + \frac{a^2}{z} \tag{3.9}$$

into the expression for the flow about a circular cylinder having a radius slightly larger than a, and so placed that the circumference passes through the point $x = a$. If, in addition, the center of the larger cylinder is placed on the x axis, the transformed curve will be that of a symmetrical wing section (Fig. 31). In the present example, let the center of the larger cylinder be placed at the point $x = -\epsilon$ where ϵ is a real quantity. The radius of this cylinder will then be $a + \epsilon$. The equation of flow about the larger cylinder with circulation is then

$$w = V\left(z^* + \epsilon + \frac{(a+\epsilon)^2}{z^* + \epsilon}\right) + \frac{i\Gamma}{2\pi} \ln \frac{z^* + \epsilon}{a + \epsilon}$$

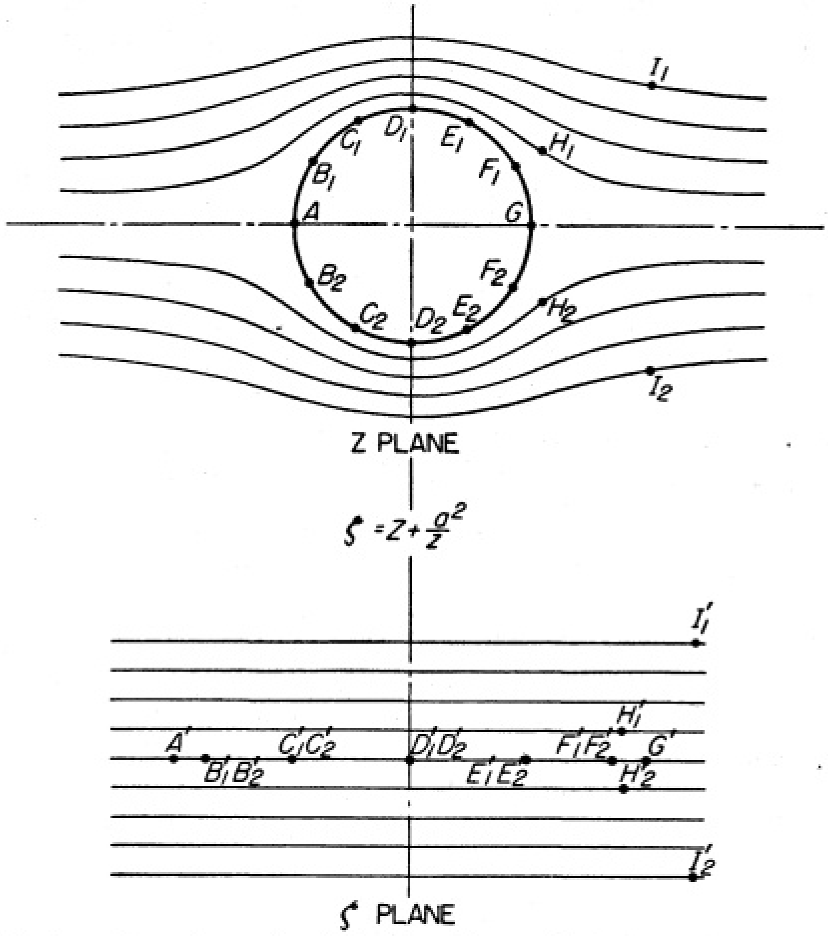

Fig. 30. Conformal transformation of the flow about a circular cylinder to uniform flow.

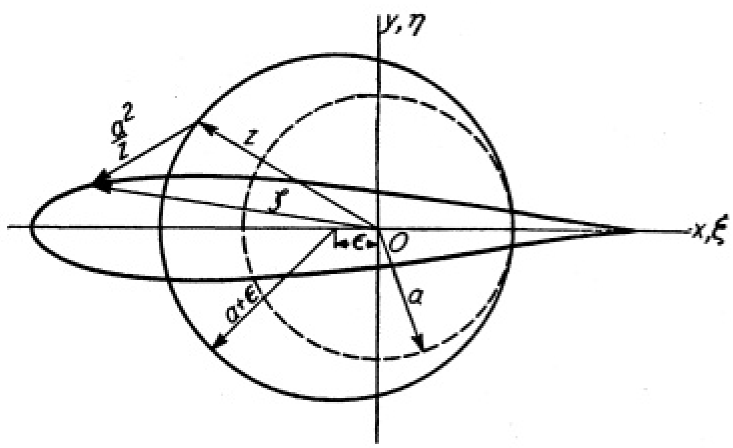

Fig. 31. Conformal transformation of a circle into a symmetrical wing section.

The more general expression for the flow about the circular cylinder with the flow at infinity inclined at an angle α_0 to the x axis is found by substituting the expression

$$z + \epsilon = (z^* + \epsilon)e^{i\alpha_0}$$

$$w = V\left[(z+\epsilon)e^{-i\alpha_0} + \frac{(a+\epsilon)^2 e^{i\alpha_0}}{z+\epsilon}\right] + \frac{i\Gamma}{2\pi}\ln\frac{(z+\epsilon)e^{-i\alpha_0}}{a+\epsilon} \qquad (3.10)$$

Substitution of Eq. (3.9) into Eq. (3.10) would result in the equation for the flow about the wing section but would lead to a complicated expression. A simple way of obtaining the shape of the wing section is to select values of z corresponding to points on the larger cylinder and find the corresponding points in the ζ plane by the use of Eq. (3.9). The velocity of any point on the wing section can be found from Eq. (3.8).

$$\frac{dw}{d\zeta} = \frac{dw}{dz}\frac{dz}{d\zeta} = \left[V\left(e^{-i\alpha_0} - \frac{(a+\epsilon)^2 e^{i\alpha_0}}{(z+\epsilon)^2}\right) + \frac{i\Gamma}{2\pi(z+\epsilon)}\right]\left(\frac{z^2}{z^2-a^2}\right)$$

It can be seen from this equation that the velocity at the point $z = a$ is infinite unless the first factor is zero. The point $z = a$ corresponds to the trailing edge of the wing section. The Kutta-Joukowsky condition states that the value of the circulation is such as to make the first factor equal to zero, which is the condition that ensures smooth flow at the trailing edge. The value of the circulation satisfying this condition is found as follows:

$$V\left[e^{-i\alpha_0} - \frac{(a+\epsilon)^2 e^{i\alpha_0}}{(a+\epsilon)^2}\right] + \frac{i\Gamma}{2\pi(a+\epsilon)} = 0$$

$$i\Gamma = 2\pi(a+\epsilon)V(e^{i\alpha_0} - e^{-i\alpha_0})$$

Since

$$\frac{e^{i\alpha_0} - e^{-i\alpha_0}}{2} = \sinh i\alpha_0 = i \sin \alpha_0$$

$$i\Gamma = 4\pi(a+\epsilon)Vi \sin \alpha_0$$

or

$$\Gamma = 4\pi(a+\epsilon)V \sin \alpha_0$$

The leading edge of the wing section corresponds to the point

$$\zeta = -a - 2\epsilon - \frac{a^2}{a+2\epsilon}$$

neglecting powers of ϵ greater than one

$$\zeta = -2a$$

Because the trailing edge corresponds to the point

$$\zeta = 2a$$

the chord of the wing section is $4a$. The lift on the wing section is $\rho V \Gamma$, and the lift coefficient is

$$c_l = 2\pi \left(1 + \frac{\epsilon}{a}\right) \sin \alpha_0$$

In the limiting case as ϵ/a approaches zero, the slope of the lift curve $dc_l/d\alpha_0$ is 2π per radian for small angles of attack. Detailed computations[37]

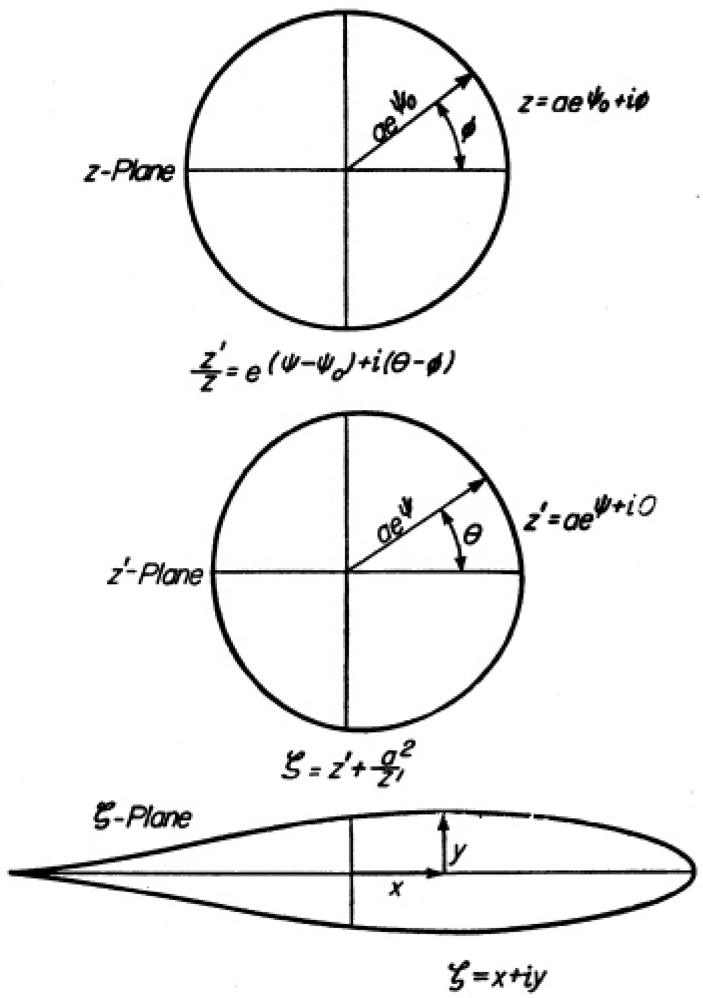

FIG. 32. Illustration of transformations used to derive airfoils and calculate pressure distributions.

will show that the thickness ratio of the wing section is nearly equal to $(3\sqrt{3}/4)(\epsilon/a)$. For wing sections approximately 12 per cent thick, the theoretical lift-curve slope is about nine per cent greater than its limiting value for thin sections.