Two Dimensional Wing and Blade Mathematical Theory

Detailing and Extending Material in Standard References

Part 4
Several Flow Fields Described Using ψ and ϕ :
#2 - The Source and Sink

Anthony Chessick, IntegEner-W, 2009

Several flow field configurations are chosen to demonstrate the use of the stream (ψ) and cross stream (ϕ) functions in describing these fields. These were chosen as flow fields in particular to be superimposed upon each other in such a way as to form a composite flow field that resembles the flow field around an airfoil section of a blade or a wing. The second is covered in this part, the flow from and to a source and sink, respectively.

Brief Review

The assumptions of flow field continuity (mass incompressibility) and irrotationality (inviscid fluid having only normal, i.e. pressure, forces acting throughout) make it possible to define mathematical function variables that realistically model actual flow. In practice, fluids such as gases will have some compressibility and even some viscosity, introducing some error into the theories obtained. At the very least, the flows that have been so defined can be said with some accuracy and confidence to extend throughout wide regions of space while having been fixed as given for only a small area within it such as the surfaces of an airfoil. With these assumptions in place, a wide range of possible flows can be investigated and conclusions reached on their impacts that come quite close to what will be found in practical applications.

The equations that must be satisfied in both Cartesian and polar coordinate systems for these two conditions to hold have been given in earlier parts of this series.

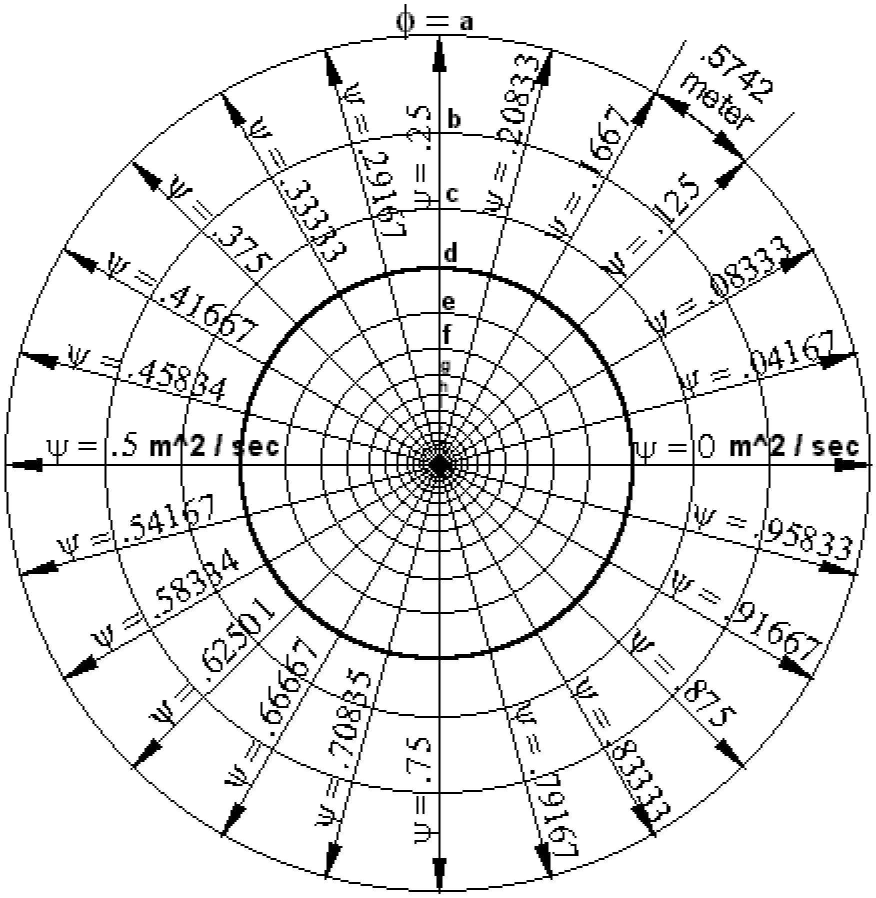
Flow Field Descriptions Necessary

The problem to be studied herein is the determination of the forces acting on an airfoil for wind energy purposes as having been provided in reference text sources for aviation purposes. Using the method of analysis that has become standardized and makes use of the stream and cross stream functions defined in Parts 1 and 2, whose gradients are equal to the flow velocities, a superposition of such functions for several flow configurations will be needed to obtain the correct flow field. These are:

1) Uniform Flow
2) Source and Sink Flow
3) Flow Doublet
4) Vortex Flow

Uniform flow was covered in Part 3. Herewith is the flow field description of the source and sink flow. Other parts will cover the remaining flow types and provide the superposition of them.

Flow From a Source



ID	r (meters)	ϕ (m^2/sec)	u' (m/sec)
a	2.1933	.1250	.0726
b	1.6881	.08333	.0943
c	1.2993	.04167	.1225
d	1.0000	0	.1592
e	.7697	-.04167	.2068
f	.5924	-.08333	.2687
g	.4560	-.1250	.3491
h	.3510	-.1667	.4536
i	.2700	-.2083	.5892
j	.2079	-.2500	.7655
k	.1600	-.2917	.9947
l	.1231	-.3333	1.293
m	.0948	-.3750	1.679
n	.0729	-.4167	2.183

The above is a drawing of the flow field. A source of fluid is at the center producing a flow "volume" - *m* - in this two dimensional representation, dimensioned as length squared per second. The flow then moves out radially with decreasing velocity as its flow area increases. Values were chosen for the flow quantities and the dimensions of the field itself as recorded both on the flow field and in the adjoining table. The cross stream circle identified as "d" was made bold to set it as having a radius of one meter and a value for the cross stream function, ϕ, of zero.

The stream and cross stream functions in both polar and Cartesian coordinates are as follows:

$$\psi = \frac{m}{2\pi}\theta = \frac{m}{2\pi}\arctan\frac{y}{x}$$

$$\phi = \frac{m}{2\pi}\ln r = \frac{m}{2\pi}\ln\sqrt{x^2 + y^2}$$

The velocities as derived from these functions in the Cartesian coordinate system are as follows:

$$u = \frac{\partial \psi}{\partial y} = \frac{\partial \phi}{\partial x} = \frac{mx}{2\pi(x^2 + y^2)}$$

$$v = -\frac{\partial \psi}{\partial x} = \frac{\partial \phi}{\partial y} = \frac{my}{2\pi(x^2 + y^2)}$$

The velocities as derived from these functions in the polar coordinate system are as follows (u' is directed radially and v' is directed tangentially counterclockwise):

$$u' = \frac{\partial \psi}{r\partial \theta} = \frac{\partial \phi}{\partial r} = \frac{m}{2\pi r}$$

$$v' = -\frac{\partial \psi}{\partial r} = \frac{\partial \phi}{r\partial \theta} = 0$$

Flow to a Sink

The flow arrows in the above drawing are changed to point toward the center and the value of *m* is taken as negative in the above relations thus changing signs as appropriate.

The Continuity and Irrotationality Conditions

It is recommended that those who wish to become familiar with these functions actually carry out the verification of the conditions of continuity and irrotationality, which are provided in Part 3. These conditions are met here for both the Cartesian and polar coordinate systems except for the center of the flow field. This exception causes the stream function to be multivalued and a jump occurs in it at the location chosen for its 0 value as may be seen above.

It should be said that when use is made of these source and sink functions in a part of this series to follow, the form in which these functions will appear will not have this exception to the continuity and irrotationality conditions within it.